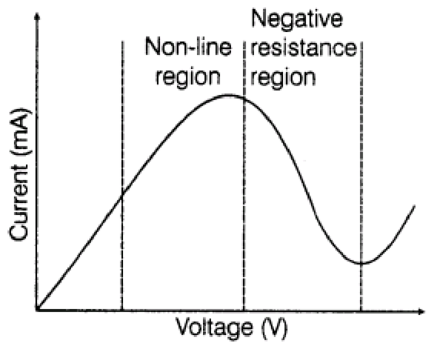
# Resistance & Ohm'sLaw

## 1 Mark Questions

1.Plot a graph showing variation of current versus voltage for the material GaAs. [Delhi 2014]

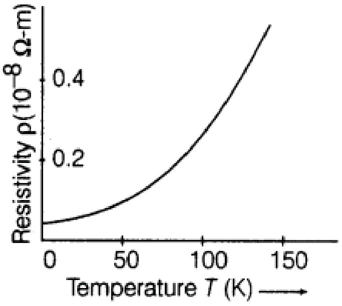
Ans.

Variation of current *versus* voltage for the material GaAs.




2.Show variation of resistivity of copper as a function of temperature in graph. [Delhi 2014; All India 2014]

**Ans**.Graph of resistivity of copper as a function of temperature is given below (resistivity of metals increases with increase in temperature)




**3.Define the term drift velocity of charge carriers in a conductor and write its relationship with the current flowing through it. [Delhi 2014]**

**Ans**.The term drift velocity of charge carriers in a conductor is defined as the average velocity acquired by the free electrons along the length of a metallic conductor under a potential difference applied across the conductor.Its relationship is expressed as

$$v_d = \frac{I}{neA}$$

where, I is current flowing through the conductor,n is concentration of free electrons
e is electron i.e.charge
A is cross-sectional area

**4.Define the term electrical conductivity of a metallic wire. Write its SI unit. [Delhi 2014]**

Ans.

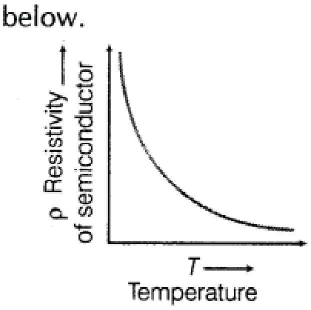
The electrical conductivity (σ) of a metallic wire is defined as the ratio of the current density to the electric field it creates. Its SI unit is mho per metre.

**5.Show variation of resistivity of Si with temperature in graph.[Delhi 2014]**

Ans.

The resistivity of a semiconductor decreases exponentially with temperature.

The variation of resistivity with temperature for semiconductor is shown in figure below.




**(1)**

**6.Define the term mobility of charge carriers in a conductor. Write its SI unit. [Delhi 2014]**

Ans.

The mobility of charge carriers in a conductor is defined as the magnitude of drift velocity (in a current carrying conductor) per unit electric field. **(1/2)**

$$\mu = \frac{\text{Drift velocity } (v_d)}{\text{Electric field } (E)} = \frac{q\tau}{m}$$

where, $\tau$ is the average relaxation time and $m$ is the mass of the charged particle.
Its SI unit is $m^2/V\text{-}s$ or $ms^{-1}N^{-1}C$. **(1/2)**

**7.Write a relation between current and drift velocity of electrons in a conductor. Use this relation to explain how the resistance of a conductor changes with the rise in temperature ? [Compartment 2013]**

**Ans**.Relation between current and drift velocity of electrons in a conductor is given by $I = Anev_d$

where I = current, A = area of conductor, n = number density of electrons and $v_d$ = drift velocity.
with the increase in temperature of a metallic conductor, resistance increases and hence, drift velocity decreases

**8.When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metal are moving in the same direction? [Delhi 2012]**

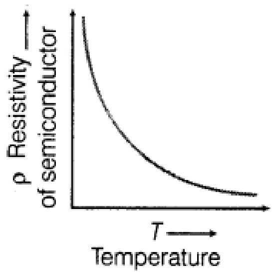
**Ans.**No, the drift speed of electrons is superposed over the random velocities of the electrons

**9.Show on a graph, the variation of resistivity with temperature for a typical semiconductor. [Delhi 2012]**

**Ans.**

The resistivity of a semiconductor decreases exponentially with temperature.

The variation of resistivity with temperature for semiconductor is shown in figure below.




**(1)**

**10.Two wires of equal length, one of copper and the other of manganin have the same resistance. Which wire is thicker? [All India 2012]**

**Ans.**

Given that resistance of both the wire is same.

i.e. $R_{Mn} = R_{Cu}$

$$\frac{\rho_{Mn} l_{Mn}}{A_{Mn}} = \rho_{Cu} \frac{l_{Cu}}{A_{Cu}} \quad \text{...(i)}$$

According to the question, both the wires are of equal length, so

$$l_{Mn} = l_{Cu}$$

∴ From Eq. (i), we get

$$\frac{\rho_{Mn}}{A_{Mn}} = \frac{\rho_{Cu}}{A_{Cu}} \text{ or } \frac{\rho}{A} = \text{constant}$$

or $$\frac{A_{Cu}}{A_{Mn}} = \frac{\rho_{Cu}}{\rho_{Mn}} \text{ or } \rho \propto A$$

We know that copper is better conductor than manganin, therefore, copper will have less resistivity.

i.e. $\rho_{Cu} < \rho_{Mn}$

So, $A_{Mn} > A_{Cu}$ $(\because \rho \propto A)$

That means wire of manganin will be thicker than that of copper. **(1)**

11.Define resistivity of a conductor. Write its SI unit. [All India 2011]

Ans.

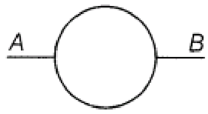
The resistivity of the material of conductor is equal to the resistance offered by the conductor of same material of unit length and unit cross-sectional area.

The resistivity of a material of the conductor does not depend on the geometry of the conductor.

SI unit of resistivity is ohm-metre ($\Omega$ -m).

**(1/2 + 1/2 = 1)**

**12.** A wire of resistance 8 $\Omega$ is bent in the form of a circle. What is the effective resistance between the ends of a diameter $AB$? **[Delhi 2010]**

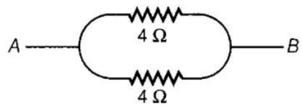



Ans.

The resistance of the whole wire is 8 Ω, which is bent in the form of a circle. We have to find the effective resistance between the ends of diameter $AB$. Diameter of the circle divides the circle into two equal parts. The resistance of each such part will be $\frac{8}{2} = 4\,\Omega$.

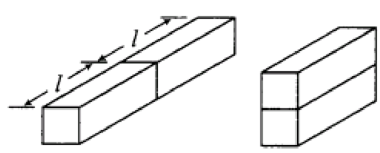
(Resistance $R \propto$ length of wire $l$, if length is halved then resistance will also become half).
From the figure, it is clear that both the parts are in parallel combination. So, effective resistance between $A$ and $B$ is given by

$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_{AB}} = \frac{1}{4} + \frac{1}{4}$$




$$\Rightarrow \quad R_{AB} = \frac{4}{1+1} = 2\,\Omega$$ **(1)**

13.Two identical slabs, of a given metal, are joined together, in two different ways, as shown in figures
(a) and (b). What is the ratio of the resistances of these two combinations? [Delhi 2010 c]




(a) (b)

Ans.

In these types of questions, first of all, identify the combination in which the metal slabs are connected and then apply the formula for equivalent resistance accordingly.

Let each conductor is of resistance $R$.

**Case I** According to Fig. (a) the resistances are connected in series combination, so equivalent resistance of slab

$$R_1 = R + R = 2R$$

**Case II** According to Fig. (b), the resistances are connected in parallel combination, so equivalent resistance

$$\frac{1}{R_2} = \frac{1}{R} + \frac{1}{R} \Rightarrow \frac{1}{R_2} = \frac{2}{R} \Rightarrow R_2 = \frac{R}{2}$$

Ratio of the equivalent resistance in two combinations is

$$\therefore \quad \frac{R_1}{R_2} = \frac{2R}{(R/2)} = 4 \Rightarrow \frac{R_1}{R_2} = 4$$ **(1)**

14.Two conducting wires X and Y of same diameter but different materials are joined in series across a battery. If the number density of electrons in X is twice than that in Y, then find the ratio of drift velocity of electrons in the two wires.[All India 2010]

Ans.

Given that number density in $X$

$= 2 \times$ Number density in $Y$

$\Rightarrow \quad n_X = 2n_Y$

As current is common for the entire circuit

i.e. $\quad I = n_X A_X \, e \, (v_d)_X = n_Y A_Y \, e \, (v_d)_Y$

Also, the diameters of the wires are same

$$A_X = A_Y$$

$$\Rightarrow \quad \frac{(v_d)_X}{(v_d)_Y} = \frac{n_Y}{n_X} = \frac{n_Y}{2n_Y} = \frac{1}{2}$$

15.The three coloured bands, on a carbon resistor are red, green and yellow, respectively. Write the value of its resistance. [All India 2009c]

Ans.

According to the colour code of resistances.

Code for red = 2

Code for green = 5

Code for yellow = 4

$\therefore$ Resistance of the wire $= 25 \times 10^4 \, \Omega \pm 20\%$

16.Write an expression for the resistivity of a metallic conductor showing its variation over a limited range of temperatures. [Delhi 2008 C]

Ans.

Required expression $\rho = \rho_0 [1 + \alpha (T_2 - T_1)]$

where, $\rho_0$ = resistivity of conductor at lower reference temperature,

$\alpha$ = temperature coefficient of resistivity,

and $\rho$ = resistivity of material of conductor. **(1)**

17.Define ionic mobility. Write its SI Unit.[Foreign 2008]

Ans.

**Ionic Mobility** The ionic mobility is the drift speed acquired by ions per unit applied electric field.

Ionic mobility,

$$\mu = \frac{v_d}{E}$$

where, $v_d$ = drift speed,

$E$ = applied electric field.

Its SI unit is $m^2/V\text{-s}$. **(1)**

**18.** A physical quantity, associated with electrical conductivity, has the SI unit $\Omega$-m. Identify this physical quantity. **[All India 2008C]**

Ans.

The physical quantity resistivity ($\rho$) of material of conductor has the SI unit $\Omega$-m. **(1)**

## 2 Marks Questions

**19.** Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7} m^2$ carrying a current of 1.5 A. Assume the density of conduction electrons to be $9 \times 10^{28} m^{-3}$. **[All India 2014]**

Ans.

Given, cross sectional area, $A = 1.0 \times 10^{-7} m^2$

Current, $I = 1.5A$

Electron density, $n = 9 \times 10^{28} m^{-3}$

Drift velocity, $v_d = ?$

We know that,

$$I = neAv_d$$ (

$$\Rightarrow v_d = \frac{I}{neA}$$

$$= \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$

$$= 1.042 \times 10^{-3} \text{ m/s}$$ (

**20.** Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $2.5 \times 10^{-7} m^2$ carrying a current of 1.8 A. Assume the density of conduction electrons to be $9 \times 10^{28} m^{-3}$. **[All India 2014]**

Ans.

Refer to ans. 19. (Ans. $5 \times 10^{-4}$ m/s).

**21.** Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $2.5 \times 10^{-7} m^2$ carrying a current of 2.7 A. Assume the density of conduction electrons to be $9 \times 10^{28} m^{-3}$. **[All India 2014]**

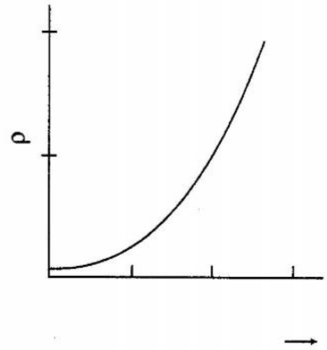
Ans.

Refer to ans. 19. (Ans. $7.5 \times 10^{-4}$ m/s).

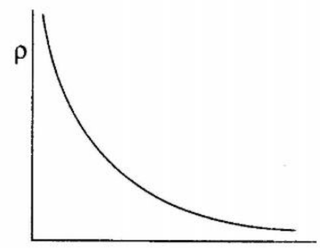
22.Draw a plot showing the variation of resistivity of a (i) conductor and (ii) semiconductor, with the increase in temperature. How does one explain this behaviour in terms of number density of charge carriers and the relaxation time? [Delhi 2014 C]

Ans.

(i) **Conductor**




(ii) **Semiconductor**



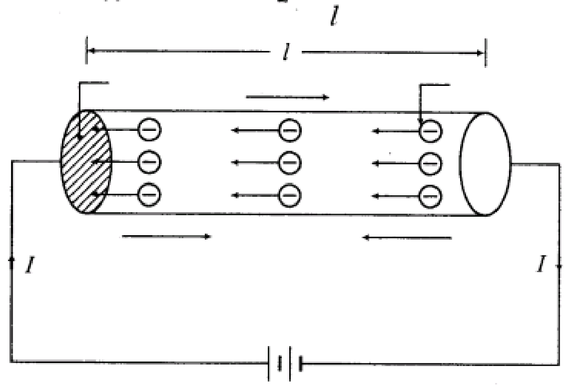

The relation between resistivity and relaxation time $\rho = \frac{m}{ne^2\tau}$

In conductors, average relation time decreases with increase in temperature, resulting in an increase in resistivity. In semiconductors, the increase in number density (with increase in temperature) is more than the decrease in relaxation time, the net result is therefore a decrease in resistivity.

**23.Derive an expression for the current density of a conductor in terms of the drift speed of electrons. [Foreign 2014]**

**Ans.**Let potential difference V is applied across a conductor of length / and hence, an electric field E produced inside the conductor.

$$\therefore \quad E = \frac{V}{l} \quad \ldots(i)$$



Let $n$ = number density of free electrons
$A$ = cross-sectional area of conductor
e = electrons charge

∴ Number of free electrons present in $l$ length of conductor = $nAl$

∴ Total charge contained in length $l$ which can contribute in current

$$q = (nAl)e \quad \text{...(i)} \; \mathbf{(1/2)}$$

The time taken by free electron to cross the $l$ length of conductor is

$$t = \frac{l}{v_d} \quad \text{...(ii)} \; \mathbf{(1/2)}$$

where, $v_d$ = drift speed of electron

∴ Current through the conductor

$$I = \frac{q}{t}$$

$$I = \frac{(nAl)\,e}{t} = \frac{(nAl)\,e}{\left(\frac{l}{v_d}\right)} = neAv_d$$

∴ Current density $(j) = \frac{I}{A} = \frac{ne\,A\,v_d}{A} = nev_d$

∴ $j = nev_d$ i.e. $j \propto v_d$) **(1)**

Thus, current density of conductor is proportional to drift speed.

24.A conductor of length / is connected to a DC source of potential V. If the length of the conductor is tripled by gradually stretching it, keeping V constant, how will
(i)drift speed of electrons and
(ii)resistance of the conductor be affected? Justify your answer.[HOTS; Foreign 2012]
Ans.

> When a wire is stretched, then there is no change in the matter of the wire hence, its volume remains constant.

**NOTE** There is a difference between the two length changed by stretching and length mode changes. Ist means that volume will not change but IInd means that volume will change.

The potential $V$ = constant, $l' = 3l$

(*i*) Drift speed of electrons $= \frac{V}{nel\rho}$

where, $n$ is number of electrons, e is charge on electron, $l$ is the length of the conductor and ρ is the resistivity of conductor.

$$\therefore \quad v \propto \frac{1}{l}$$

(∵ Other factors are constants)

So, vhen length is tripled, drift velocity gets one-third. **(1)**

(*ii*) Resistance of the conductor is given as

$$R = \rho\frac{l}{A}$$

Here, wire is stretched to triple its length, that means the mass of the wire remains same in both the conditions.

Mass before stretching = Mass after stretching

(Volume × Density) before stretching

= (Volume × Density) after stretching.

(Area of cross-section × Length) before stretching

= (Area of cross-section × Length) after stretching

(∵ Density is same in both cases)

$$\therefore \quad A_1 l_1 = A_2 l_2$$

$$A_1 l = A_2 (3l)$$

(∵ Length is tripled after stretching)

$$\therefore \quad A_2 = \frac{A_1}{3}$$

i.e. when length is tripled area of cross-section is reduced to $\frac{A}{3}$.

Hence, $R = \rho \frac{l'}{A'} = \rho \frac{3l}{\frac{A}{3}} = 9\rho \frac{l}{A} = 9R$ **(1)**

Thus, new resistance will be 9 times of its original value.

25.Plot a graph showing temperature dependence of resistivity for a typical semiconductor. How is this behaviour explained? [Delhi 2011]

Ans.

To plot the graph between the two quantities, first of all identify the relation between them.

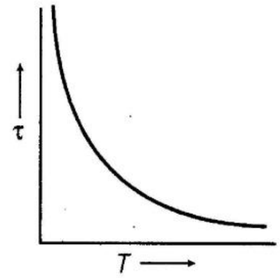
Since, resistivity of material of conductor (ρ) is given by

$$\rho = \frac{m}{ne^2\tau}$$

where, $n$ = number density of electrons,

$\tau$ = relaxation time.

With the rise of temperature of semiconductor, number density of free electrons increases, whereas $\tau$ remains constant and hence resistivity decreases. **(1)**




Resistivity of a semiconductor decreases rapidly with temperature

**(1)**

**26.** (*i*) You are required to select a carbon resistor of resistance 47 k $\Omega \pm 10\%$ from a large collection. What should be the sequence of colour bands used to code it?

(*ii*) Write two characteristics of manganin which make it suitable for making standard resistances.

**[Delhi 2011]**

Ans.

(*i*) Given, resistance $= 47\,k\Omega \pm 10\%$

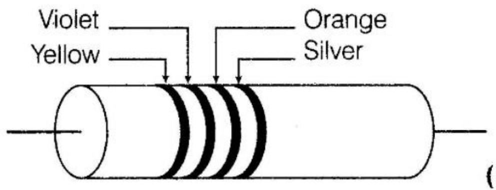
$$= 47 \times 10^3\ \Omega \pm 10\%$$

$\therefore$ Ist colour band should be yellow as code for it is 4.

IInd colour band should be violet as code for it is 7.

IIIrd colour band should be orange as code for it is 3.

IVth colour band should be silver because approximation is $\pm 10\%$




**(1)**

(*ii*) Two properties of manganin are

(*a*) Low temperature coefficient of resistance.

(*b*) High value of resistivity of material of manganin make it suitable for making a standard resistor. **(1)**

27.The sequence of coloured bands in two carbon resistors $R_t$ and $R_2$ is

(i)brown, green, blue and

(ii)orange, black, green.

Find the ratio of their resistances.[Delhi 2010 C]

Ans.

According to colour codes, resistance of two wires are

(*i*) Code of brown = 1

Code of green = 5

Code of blue = 6

$R_1 = 15 \times 10^6\,\Omega \pm 20\%$

(*ii*) Code of orange = 3

Code of black = 0

Code of green = 5

$R_2 = 30 \times 10^5\,\Omega \pm 20\%$

$\therefore$ Ratio of resistances $\frac{R_1}{R_2} = \frac{15 \times 10^6}{30 \times 10^5} = 5$

$\Rightarrow \qquad \frac{R_1}{R_2} = 5$

**(1)**

**28.** A wire of 15 Ω resistance is gradually stretched to double its original length. It is then cut into two equal parts. These parts are then connected in parallel across a 3.0 V battery. Find the current drawn from the battery. **[All India 2009]**

Ans.

Let original cross-sectional area and length of 15 Ω resistance are $A$ and $l$ after stretching they become $A'$ and $l'$, respectively.

Initial resistance, $R = \rho \frac{l}{A} \Rightarrow 15 = \rho \frac{l}{A}$ ...(i)

∴ In case of stretching, volume of the wire remains same, so

$$Al = A'l'$$

∵ $l' = 2l \Rightarrow A' = \frac{A}{2}$ ...(ii)

∴ Resistance after stretching

$$R' = \rho \frac{l'}{A'} = \rho \left(\frac{2l}{A/2}\right) = 4\left(\rho \frac{A}{l}\right)$$

$R' = 4 \times 15$ (from Eq. (i))

Now resistance, $R' = 60\ \Omega$ **(1/2)**

After dividing into two parts, resistance of each part = 30 Ω

∴ Effective resistance after connecting them into parallel combination

$R_{eff} = \frac{30}{2} = 15\ \Omega$ **(1/2)**

∵ Applied potential difference, $V = 3$ V

∴ Current drawn from the battery, $I = \frac{V}{R}$

(from Ohm's law)

⇒ $I = \frac{3}{15} \Rightarrow I = \frac{1}{5}$ A **(1)**

29.Derive an expression for drift velocity of free electrons in a conductor in terms of relaxation time. [Delhi 2009]

Ans.

When a conductor is subjected to an electric field **E**, each electron experiences a force

$\mathbf{F} = -e\mathbf{E}$, and free electron acquires an acceleration

$$\mathbf{a} = \frac{\mathbf{F}}{m} = -\frac{e\mathbf{E}}{m} \quad \text{...(i)}$$

where, $m$ = mass of electron, e = electronic charge and **E** = electric field.

Free electron starts accelerating and gains velocity and collide with atoms and molecules of the conductor. The average time difference between two consecutive collisions is known as relaxation time of electron and

$$\bar{\tau} = \frac{\tau_1 + \tau_2 + \ldots + \tau_n}{n} \quad \text{...(ii)} \quad \textbf{(1)}$$

where, $\tau_1, \tau_2, \ldots, \tau_n$ are the average time difference between 1st, 2nd, ..., $n$th collisions.

$\therefore$ $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$, are velocities gained by electron in 1st, 2nd ..., $n$th collisions with initial thermal velocities $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n$, respectively.

$$\therefore \quad \mathbf{v}_1 = \mathbf{u}_1 + \mathbf{a}\tau_1$$
$$\mathbf{v}_2 = \mathbf{u}_2 + \mathbf{a}\tau_2$$
$$\vdots \quad \vdots \quad \vdots$$
$$\mathbf{v}_n = \mathbf{u}_n + \mathbf{a}\tau_n$$

The drift speed $\mathbf{v}_d$ may be defined as

$$\mathbf{v}_d = \frac{\mathbf{v}_1 + \mathbf{v}_2 + \ldots + \mathbf{v}_n}{n}$$

$$\mathbf{v}_d = \frac{(\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n) + \mathbf{a}(\tau_1 + \tau_2 + \ldots + \tau_n)}{n}$$

$$\mathbf{v}_d = \frac{(\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n)}{n} + \frac{\mathbf{a}(\tau_1 + \tau_2 + \ldots + \tau_n)}{n}$$

$$\mathbf{v}_d = 0 + \mathbf{a}\tau \quad (\because \text{Average thermal velocity in } n \text{ collisions} = 0)$$

$$\mathbf{v}_d = -\left(\frac{e\mathbf{E}}{m}\right)\tau \quad \text{(from Eq. (i))}$$

**(1)**

This is the required expression of drift speed of free electrons.

30.Two metallic wires of the same material have the same length but cross-sectional area is in the ratio 1: 2. They are connected

(i)in series and

(ii)in parallel.

Compare the drift velocities of electrons in the two wires in both the cases. [All India 2008]

Ans.

In series, current is same, $I = neAv_d$

$$v_d = \frac{I}{neA}$$

$$\Rightarrow \quad v_d \propto \frac{I}{A}$$

$$\frac{v_{d_1}}{v_{d_2}} = \frac{A_2}{A_1} = \frac{2}{1}$$

In parallel, voltage is same.

$$\text{As, } v_d = \frac{e\tau}{m}\tau = \frac{ev}{ml}\tau, \; v_d \propto \frac{1}{l}$$

$$\frac{v_{d_1}}{v_{d_2}} = \frac{l_2}{l_1} = \frac{1}{1}$$

31.Derive an expression for the resistivity of a good conductor, in terms of the relaxation time of electrons. [All India 2008]

Ans.

When a potential difference $V$ is applied across $l$ length of a conductor then drift speed of electron is given by

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{lm} \quad \text{...(i)} \quad \left(\because E = \frac{V}{l}\right)$$

**(1/2)**

Also, the electric current through the conductor and drift speed are linked as

$$I = neAv_d \quad \text{...(ii)}$$ **(1/2)**

where, $n$ = number density of electrons

$e$ = electronic charge

$A$ = cross-sectional area of conductor

$v_d$ = drift speed of electron

$$\therefore \quad I = neA\left(\frac{eV\tau}{lm}\right)$$

(substituting the value of $v_d$)

$$\Rightarrow \quad \frac{V}{I} = \frac{ml}{ne^2\tau A} \quad \text{...(iii)}$$

Also, at constant temperature

$$\frac{V}{I} = R \quad \text{...(iv)}$$

(from Ohm's law)

$$\Rightarrow \quad R = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A}$$

(from Eqs. (iii) and (iv))

But $\quad R = \rho\frac{l}{A}$

where, $\rho$ is specific resistance or resistivity of conductor.

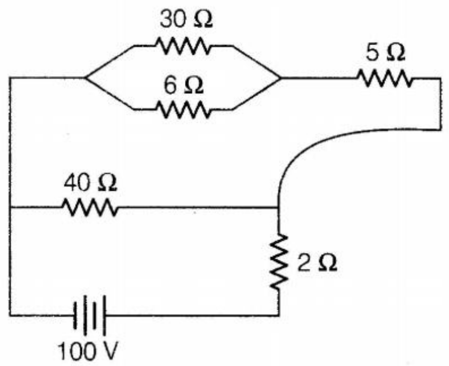
$$\therefore \quad \rho = \frac{m}{ne^2\tau} \quad \text{...(v)}$$ **(1)**

Thus, resistivity of material of conductor is inversely proportional to relaxation time ($\tau$).

## 3 Marks Questions

**32.** (a) Define the term 'drift velocity' of charge carriers in a conductor. Obtain the expression for the current density in terms of relaxation time.

(b) A 100 V battery is connected to the electric network is shown in the figure. If the power consumed in the $2\,\Omega$ resistor is 200 W, determine the power dissipated in the $5\,\Omega$ resistor. **[Foreign 2014]**




**Ans**.(i)The term drift velocity of charge carriers in a conductor is defined as the average velocity acquired by the free electrons along the length of a metallic conductor under a potential difference applied across the conductor.Its relationship is expressed as

$$v_d = \frac{I}{neA}$$

where, I is current flowing through the conductor,n is concentration of free electrons
e is electron i.e.charge
A is cross-sectional area

Let original cross-sectional area and length of $15\,\Omega$ resistance are $A$ and $l$ after stretching they become $A'$ and $l'$, respectively.

Initial resistance, $R = \rho\dfrac{l}{A} \Rightarrow 15 = \rho\dfrac{l}{A}$ ...(i)

$\therefore$ In case of stretching, volume of the wire remains same, so

$$Al = A'l'$$

$\because \quad l' = 2l \Rightarrow A' = \frac{A}{2}$ ...(ii)

$\therefore$ Resistance after stretching

$$R' = \rho\frac{l'}{A'} = \rho\left(\frac{2l}{A/2}\right) = 4\left(\rho\frac{A}{l}\right)$$

$R' = 4 \times 15$ (from Eq. (i))

Now resistance, $R' = 60\ \Omega$ **(1/2)**

After dividing into two parts, resistance of each part $= 30\ \Omega$

$\therefore$ Effective resistance after connecting them into parallel combination

$R_{eff} = \frac{30}{2} = 15\ \Omega$ **(1/2)**

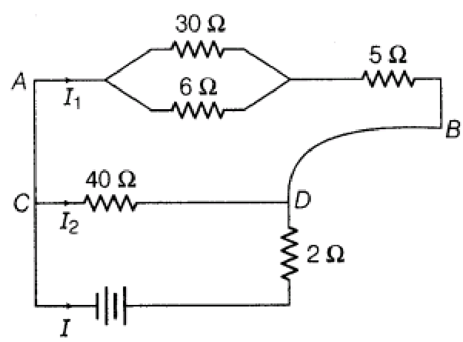
$\because$ Applied potential difference, $V = 3$ V

$\therefore$ Current drawn from the battery, $I = \frac{V}{R}$

(from Ohm's law)

$\Rightarrow \quad I = \frac{3}{15} \Rightarrow I = \frac{1}{5}$ A **(1)**

(ii) Total current, $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{200}{2}} = 10$A




Resistance across $AB$

$$= \left(\frac{1}{30} + \frac{1}{6}\right) + 5$$

$R_{AB} = 10\ \Omega = R_1$

Potential difference across $AB$ = potential difference across $CD$

$\Rightarrow \quad I_1R_1 = I_2R_2 \Rightarrow I_1R_1 - I_2R_2 = 0$

$\therefore \quad 10I_1 - 40I_2 = 0$

$I_1 - 4I_2 = 0$ ...(i)

$\because \quad I_1 + I_2 = I \Rightarrow I_1 + I_2 = 10$ ...(ii)

From Eq. (i)

$I_2 = \frac{I_1}{4}$ ...(iii)

Put the value of $I_2$ from Eq. (iii) to Eq. (ii), we get

$$I + \frac{I_1}{4} = 10 \Rightarrow I_1 = 8A$$

Power dissipated in 5 Ω resistor

$= 5 \times$ [(current through 5 Ω resistor)$^2$]

$= 5 \times (8)^2$

$P = 320$ W

33.Define relaxation time of the free electrons drifting in a conductor. How it is related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material.[All India 2012]

Ans.

**Relaxation Time** The average time difference between two successive collisions of drifting electrons inside the conductor under the influence of electric field applied across the conductor, is known as relaxation time. **(1)**

Drift speed and relaxation time

$$v_d = -\frac{eE\tau}{m} \qquad \textbf{(1/2)}$$

where, $E =$ electric field due to applied potential difference

$\tau =$ relaxation time

$m =$ mass of electron

$e =$ electronic charge

$\because$ Electron current, $I = -neAv_d$ **(1/2)**

$$I = -neA\left(-\frac{eE\tau}{m}\right) \qquad \textbf{(1/2)}$$

$$I = \frac{ne^2A\tau}{m}\left(\frac{V}{l}\right) \qquad \left(\because E = \frac{V}{l}\right)$$

$$\Rightarrow \quad \frac{V}{I} = \frac{mI}{ne^2A\tau} = \rho\frac{l}{A} = R$$

$$\Rightarrow \quad \rho = \frac{m}{ne^2\tau} \qquad \textbf{(1/2)}$$

This is required expression.

**34.** (*i*) Derive the relation between current density $j$ and potential difference $V$ across a current carrying conductor of length $l$, area of cross-section $A$ and the number density $n$ of free electrons.

(*ii*) Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7}\ m^2$ carrying a current of 1.5 A. [Assume that the number density of conduction electrons is $9 \times 10^{28}\ m^{-3}$]. **[Delhi 2012 C]**

Ans.

(*i*) The current in the conductor having length $l$, cross-sectional area $A$ and number density $n$ is

$$I = neAv_d \qquad \text{...(i)}$$

Electric field inside the wire is given by

$$E = \frac{V}{l} \qquad \text{...(ii)}$$

If relaxation time is $\tau$, the drift speed

$$v_d = \frac{e\tau E}{m} \quad \textbf{(1)}$$

where,

$m$ = mass of electron

$\tau$ = relaxation charge

$e$ = electronic charge

$E$ = electric field.

Put the value of Eq. (i), we get

$$\Rightarrow \quad I = \frac{ne^2\tau}{m} AE \quad \ldots\text{(ii)}$$

From Eqs. (ii) and (iii), we get

$$I = \frac{ne^2\tau AV}{ml} \Rightarrow J = \frac{I}{A} = \frac{ne^2\tau V}{ml} \quad \textbf{(1)}$$

(*ii*) Given, $I = 1.5$ A, $n = 9 \times 10^{28}$ m$^{-3}$,

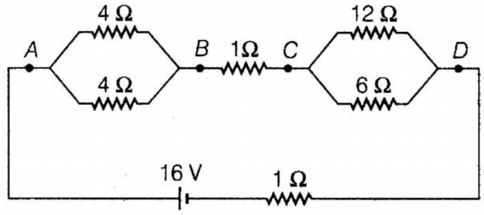
$$A = 1.0 \times 10^{-7} \text{ m}^2$$

$$\because \quad v_d = \frac{I}{neA}$$

$$\therefore v_d = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$

$$\Rightarrow v_d = 1.04 \times 10^{-3} \text{ m/s} \quad \textbf{(2)}$$

**35.** A network of resistors is connected to a 16 V battery of internal resistance of 1 Ω as shown in the figure.




(*i*) Compute the equivalent resistance of the network.

(*ii*) Obtain the voltage drops $V_{AB}$ and $V_{CD}$. **[Foreign 2010]**

Ans.

To calculate the equivalent resistance of complex network (network having multiple branches), calculate the equivalent resistance of smaller part of network and finally calculate the equivalent resistance of the network.

(*i*) $\because$ 4 Ω and 4 Ω are in parallel combination.

$\therefore$ Equivalent resistance, $R_{AB} = \frac{1}{4} + \frac{1}{4}$

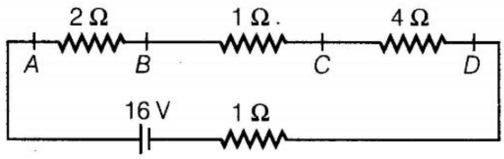
$$\frac{1}{R_{AB}} = \frac{2}{4} \Rightarrow R_{AB} = 2\,\Omega$$

Similarly, equivalent resistance of 12 Ω and 6 Ω is

$$\frac{1}{R_{BC}} = \frac{1}{12} + \frac{1}{6} \Rightarrow \frac{1}{R_{BC}} = \frac{1+2}{12}$$

$$\Rightarrow \quad R_{BC} = 4\,\Omega$$

Now, the circuit can be redrawn as shown in figure below




Now, 2 Ω, 1Ω and 4Ω, 1Ω are in series combination.

$\therefore$ Equivalent resistance of the network

$R_{eq} = 2\,\Omega + 1\Omega + 4\,\Omega + 1\Omega = 8\,\Omega$ **(1)**

(*ii*) $\therefore$ Current drawn from the battery,

$$I = \frac{V}{R} = \frac{16}{8} = 2\text{ A}$$

This current will flow from *A* to *B* and *C* to *D*. So, the potential difference in between *AB* and *CD* can be calculated as

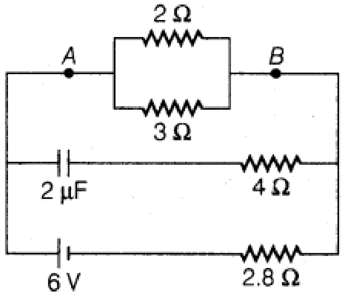
Now,

$V_{AB} = IR_{AB} = 2 \times 2 = 4\text{ V}$ **(1)**

and $V_{CD} = I\,R_{CD} = 2 \times 2 = 8\text{ V}$ **(1)** **3**

**36.** Calculate the steady current through the 2 Ω resistor in the circuit shown in the figure.

**[HOTS; Foreign 2010]**




Ans.

> To calculate the current through a particular resistance first we have to find the potential difference across that resistance.

In DC circuit, capacitor offers infinite resistance. Therefore, no current flows through capacitor and through 4 Ω resistance, so resistance will produce no effect.

∴ Effective resistance between $A$ and $B$

$$R_{AB} = \frac{2 \times 3}{2+3} = 1.2\ \Omega$$

$$\left(\because \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}\right) \quad \textbf{(1)}$$

Total resistance of the circuit = 1.2 + 2.8 = 4 Ω

(∵ These two are in series)

Net current drawn from the cell,

$$I = \frac{V}{R\,(\text{total resistance})}$$

$$= \frac{6}{4} = \frac{3}{2} = 1.5\ \text{A} \quad \textbf{(1/2)}$$

∴ Potential difference between $A$ and $B$

$$V_{AB} = IR_{AB} = 1.5 \times 1.2$$

$$V_{AB} = 1.80\ \text{V} \quad \textbf{(1/2)}$$

Current through 2 Ω resistance,

$$I' = \frac{V_{AB}}{2\ \Omega} = \frac{1.8}{2} = 0.9\ \text{A} \quad \textbf{(1)}$$

37.Three resistors $R_1$, $R_2$ and $R_3$ are connected in parallel, across a source of emf E and negligible internal resistance. Obtain a formula for the equivalent expressions for the current through each of the three resistors. [All India 2009 c]

Ans.

Let the equivalent resistance of parallel combination of $R_1$, $R_2$ and $R_3$ is $R$.

$$\therefore \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}$$

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1} \quad \left(1\frac{1}{2}\right)$$

Effective current, $I = \dfrac{E}{R}$

$$I = \frac{E}{\dfrac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}}$$

$$I = \frac{E(R_1R_2 + R_2R_3 + R_3R_1)}{R_1R_2R_3} \quad \left(1\frac{1}{2}\right)$$

38.Prove that the current density of a metallic conductor is directly proportional to the drift speed of electrons.[Delhi 2008]

Ans.

Since, the relationship between electric current density ($j$) and drift velocity ($v_d$) is given by

$$j = nev_d \quad \text{...(i)} \quad \textbf{(2)}$$

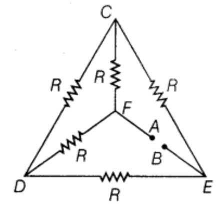
where, $n$ = number of free electrons per unit volume, e = charge on each electron

For detailed proof, refer ans 31.

$\because$ For a given conductor $ne$ is constant.

$\therefore \quad j \propto v_d$ **(1)**

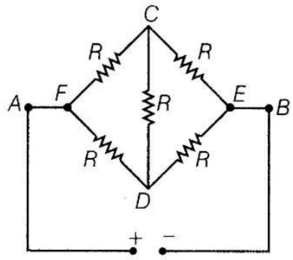
**39.** (*i*) Calculate the equivalent resistance of the given electrical network between points *A* and *B*.




(*ii*) Also, calculate the current through *CD* and *ACB*, if a 10 V DC source is connected between *A* and *B* and the value of *R* is assumed as 2 Ω. **[All India 2008]**

Ans.

(*i*) The circuit can be redrawn as shown in figure below




**(1)**

As combination is balanced Wheatstone bridge.

$\therefore$ Equivalent resistance between the points *A* and *B*

$$R_{eq} = \frac{2R \times 2R}{2R + 2R} = R$$ **(1)**

(*ii*) If $R = 2\Omega$, then $R_{eq} = 2\ \Omega$

There is no current through resistor across *CD*.

$\therefore$ Current through arm $AFCEB = \frac{V}{2R}$

$$= \frac{10}{2 \times 2} = 2.5\ \text{A}$$ **(1)**